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Introduction to "Numerical Simulation of Hydrodynamics by the Method of Point Vortices"

When I was on sabbatical at Oxford in 1972, Keith Roberts invited me to consult with his group at Culham Laboratories. On my first visit two things impressed me: Roberts' pioneering vision about computational fluid and plasma dynamics and the environment required to facilitate productivity, and Jess Christiansen's robust code for two-dimensional inviscid incompressible homogeneous flows.

Jess welcomed me with copies of his green-covered Culham reports, one being the paper being introduced here and the other being "Vortex: A two dimensional hydrodynamics visualization code," a detailed manual with flow charts and code published in July 1970. He also showed me his plots of many classic hydrodynamic configurations, where point vortices evolved on 2d domains resolved by equispaced meshes.

The particle-mesh idea came from the plasma physics community. However, Jess (with Keith) noted (p. 365), "One reason why vortices are important in hydrodynamics is that they are the *sources* for the incompressible flow field ... the current state of the system and its future evolution are in principle determined subject to appropriate boundary conditions."

The particles represented the point vortices with constant positive and negative circulation and for convenience the circulation magnitudes were the same. The projection of particles to the mesh and the calculation of velocities of particles from the stream function obtained by inverting Poisson's equation on the mesh, struck me as most novel. This dual process is the essential *dispersive* regularization step that makes the code robust and gives rise to the characteristic numerical small-scale structures associated with the high gradients of vorticity implicit in these representations. He discusses the so called "nearest-grid-point" and the preferred "cloud-in-cell" or area weighting ideas, namely a second-order accurate interpolation.

The initializations were all of piecewise constant vortex regions, circular or rectangular in shape (i.e., layers), each filled with regularly spaced point vortices. The boundary conditions were periodic or fixed. It was noted that the "circular" vortex developed $m = 4$ waves on the surface as a result of interacting with image vortices that arise from the boundary conditions.

Even at that time a form of "parallelization" was used to increase capacity. A prominent feature of the code was that each word of memory was packed with both the x and y positions and incremental change due to the particle advection. That is, integer arithmetic was used on words where each coordinate was represented by 6 bits for the exponent and 18 bits for the mantissa.

The validity of phenomena observed was discussed in terms of the growth of anomalous modes on the vortex boundary. Note the comment (p. 373) "in the study of the interaction between vortices {24} the picture of evolution is almost complete within 3, 4, or 5 periods of rotation of a single vortex." In later papers with J. B. Taylor the variation of energy and correlation functions was also given.

This paper brought forth new insights for the era when weak nonlinearity or short-time solutions broke new pathways. For example, because of recent work with Gary Deem [1] and my presence, Jess and I began to collaborate on the finite area vortex region model of the von Karman wake in the linear and nonlinear time regions. In this work [2], we were able to see the wave motions induced on the surface of vortices by nearby vortices (elliptical, $m = 2$, forms and "breaking" and filamentation forms when the interactions were weak and strong, respectively).

The insights gleaned also led me, Keith Roberts, Mike Hughes, and Gary Deem to generalize the "water-bag" model of Berk and Roberts (plasma physicists) to the "contour dynamics" model for the two-dimensional Euler equations [3, 4] and later to the ionospheric cloud "gradient-drift" instability [5].

Much was left for the next generations of computational fluid physicists. A citation index search shows that the code inspired work on point vortex representations of the Navier–Stokes equations and boundary layer phenomena in two dimensions and a host of simulations of the flow around cylinders as well as applications in geophysical fluid dynamics. Some of these references are given in [6, 7].

REFERENCES

1. N. J. Zabusky and G. S. Deem, Dynamical evolution of two-dimensional unstable shear flows, *J. Fluid Mech.* **47,** 353 (1971).

0021-9991/97 $25.00

2. N. J. Zabusky and J. P. Christiansen, Instability, coalescence, and fission of finite-area vortex structures, *J. Fluid Mech.* **61,** 219 (1973).
3. G. S. Deem and N. J. Zabusky, Vortex waves: Stationary 'V-states,' interactions, recurrence and breaking, *Phys. Rev. Lett* **40,** 859 (1978).
4. N. J. Zabusky, M. Hughes, and K. V. Roberts, Contour dynamics for the Euler equations in two dimensions, *J. Comput. Phys.* **30,** 96 (1979).
5. E. A. Overman II and N. J. Zabusky, Stability and nonlinear evolution of plasma clouds via regularized contour dynamics, *Phys. Rev. Lett* **45,** 1693 (1980).
6. A. Leonard, Vortex methods for flow simulation, *J. Comput. Phys.* **37,** 289 (1980).
7. P. Koumoutsakos, A. Leonard, and F. Pepin, Boundary conditions for viscous vortex methods, *J. Comput. Phys.* **113,** 52 (1994).

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